Precalculus with Trigonometry

Lesson 4-3 · Set 6 Mini Exploration Trigonometric Function Properties and Identities Name: Date:

Problem 1

Quick! Without looking at any of your notes, complete these properties:

$$\frac{1}{\tan x} =$$
$$\frac{1}{\sin x} =$$
$$\frac{1}{\cos x} =$$

Okay, look back at your notes, ask a friend, or check the Interwebz... However you want, confirm your answers above. (And fix, if necessary.)

Problem 2

Quick! Complete what's missing:

$$()(\csc x) = 1$$

 $(\cot x)() = 1$
 $()(\sec x) = 1$

Unsure about one of your answers? You can check with a table in Desmos. Correct? It'll look like this:

x_1	$(((csc x_1)$
1	1
2	1
3	1
4	1
5	1

Incorrect? It'll look something like this:

$(((csc x_1)$
0.64209262
-0.45765755
-7.0152526
0.86369115
-0.29581292

Problem 3

A few more properties for review... Write these in terms of sine and cosine:

 $\tan x =$

 $\cot x =$

Problem 4

Last round. Fill in what's missing:

$$\cos^{2} x + \sin^{2} x = ($$
)
$$1 + ($$
) = sec² x
$$\cot^{2} x + 1 = ($$
)

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Problem 5

Here's a new style of identity:

 $\left(\cos\phi - \sin\phi\right)^2 = 1 - 2\cos\phi\sin\phi$

Which side would you start with? Why? Write something down. Then discuss your reasoning with a partner.

If you noticed that the only difference is cosine squared and sine squared vs 1, you're on track. I seem to recall that one of our most useful properties involves cosine squared and sine squared. Let's rewrite the expression so those terms are adjacent:

 $=\cos^2\phi + \sin^2\phi - 2\cos\phi\sin\phi$

And since $\cos^2 x + \sin^2 x = 1$, we get:

 $= 1 - 2\cos\phi\sin\phi$

And we're done!

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 $\therefore (\cos\phi - \sin\phi)^2 = 1 - 2\cos\phi\sin\phi, \Box$

Problem 7

Time for you to try one. Prove:

Problem 6

Okay, same identify as #5:

 $(\cos\phi - \sin\phi)^2 = 1 - 2\cos\phi\sin\phi$

Let's focus on the left. What could we do? Well, for starters, we could write out those two factors. And then...? Multiply! Here goes:

 $(\cos\phi - \sin\phi)^2 = (\cos\phi - \sin\phi)(\cos\phi - \sin\phi)$ $= \cos^2\phi - \cos\phi\sin\phi - \sin\phi\cos\phi + \sin^2\phi$

What's next? Well, simplifying wouldn't hurt. The middle two terms are like terms, so we get:

 $=\cos^2\phi - 2\cos\phi\sin\phi + \sin^2\phi$

Okay, stop. What are our options now? One idea is to compare what we have with our final target.

We have: $\cos^2 \phi - 2\cos \phi \sin \phi + \sin^2 \phi$

Our goal: $1 - 2\cos\phi\sin\phi$

Describe at least one similarity and at least one difference that you see.

$$(1 - \tan \phi)^2 = \sec^2 \phi - 2 \tan \phi$$